

Swarm Extended Kalman Filter

For a nonlinear system:

$$\dot{x}(t) = F(x,t) + G(x,t)w(t)$$

$$y(t) = h(x,t) + v(t)$$

with states $x(t) =$

\dot{v}	orb frame acceleration scalar
v	orb frame velocity scalar
$\dot{\phi}$	side tilt rate
ϕ	side tilt angle
$\dot{\theta}$	front tilt rate
θ	front tilt angle
ψ	heading
x	= position East
y	= position North
x_{ab}	= x accelerometer bias
y_{ab}	y " "
z_{ab}	z " "
x_{rb}	x rate gyro bias
y_{rb}	y " "
z_{rb}	z " "

& measurements $y(t) =$

z_a	x accelerometer
y_a	y " "
z_a	z " "
z_r	x rate gyro
z_r	z rate gyro
z_g	x GPS
y_g	y " "
z_g	z " "
ψ_g	heading "
v_g	velocity scalar from GPS
w	wheel speed

Nonlinear System $\dot{x}(t) = F(x, t) + G(x, t)w(t)$

$F(x, t) =$

$$\ddot{v} = 0$$

$$\dot{v} = \dot{v}$$

$$\ddot{\theta} = 0$$

$$\dot{\theta} = \dot{\theta}$$

$$\ddot{\alpha} = 0$$

$$\dot{\alpha} = \dot{\alpha}$$

$$\dot{\psi} = \frac{v}{r} \tan \beta$$

$$\dot{x} = v \cos \psi$$

$$\dot{y} = v \sin \psi$$

$$\dot{x}_{ob} = 0$$

$$\dot{y}_{ob} = 0$$

$$\dot{z}_{ob} = 0$$

$$\dot{x}_{rb} = 0$$

$$\dot{z}_{rb} = 0$$

Measurements $y(t) = h(x, b) + v(t)$

$$\begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix} = {}^C A R \begin{bmatrix} \dot{x} \\ \dot{y} \\ -g \end{bmatrix}$$

$$\text{where } {}^C A R = \begin{bmatrix} c\psi c\theta + s\psi s\phi s\theta & -s\psi c\theta + c\psi s\phi s\theta & -c\phi s\theta \\ s\psi c\theta & c\psi s\theta & s\phi \\ c\psi s\theta - s\psi s\phi c\theta & -s\psi s\theta - c\psi s\phi c\theta & c\phi c\theta \end{bmatrix}$$

$$\dot{x} = \dot{v} \cos\psi - \frac{v^2}{r} \tan\phi \sin\psi$$

$$\dot{y} = \dot{v} \sin\psi + \frac{v^2}{r} \tan\phi \cos\psi$$

$$g = 9.81 \text{ m/s}^2$$

no sensor $\rightarrow \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + {}^C B R \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + {}^C A R \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}$

$$\text{where } {}^C B R = \begin{bmatrix} c\theta & s\phi s\theta & -c\phi s\theta \\ 0 & c\phi & s\phi \\ s\theta & -s\phi c\theta & c\phi c\theta \end{bmatrix}$$

$$\text{so } x_r = c\theta \dot{\phi} - c\phi s\theta \dot{\psi}$$

$$z_r = s\theta \dot{\phi} + c\phi c\theta \dot{\psi}$$

GPS $x_g = x$

$$y_g = y$$

$$\psi_g = \psi$$

$$v_g = v$$

Encoder $\omega = \frac{v}{r \cos\phi}$

Note: $c\theta$ means $\cos\theta$, $s\theta$ means $\sin\theta$, etc.